

$$z = x^2 + y^2 - 1 \quad x^2 + 4y^2 - 2x = 0$$

$$L(x, y, \lambda) = x^2 + y^2 - 1 + \lambda(x^2 + 4y^2 - 2x)$$

$$L(x, y, \lambda) = x^2 + y^2 - 1 + \lambda x^2 + 4\lambda y^2 - 2\lambda x$$

$$L'_x = 2x + 2\lambda x - 2\lambda$$

$$L'_y = 2y + 8\lambda y$$

$$L'_{\lambda} = x^2 + 4y^2 - 2x$$

$$\begin{cases} 2x + 2\lambda x - 2\lambda = 0 \\ 2y + 8\lambda y = 0 \\ x^2 + 4y^2 - 2x = 0 \end{cases}$$

$$\begin{cases} x + \lambda x - \lambda = 0 \\ y + 4\lambda y = 0 \\ x^2 + 4y^2 - 2x = 0 \end{cases}$$

$$\begin{cases} x + \lambda x - \lambda = 0 \\ y(1+4\lambda) = 0 \\ x^2 + 4y^2 - 2x = 0 \end{cases}$$

$$\begin{cases} x + \lambda x - \lambda = 0 \\ y = 0 \\ x^2 + 4y^2 - 2x = 0 \end{cases}$$

$$\begin{cases} x + \lambda x - \lambda = 0 \\ 1 + 4\lambda = 0 \\ x^2 + 4y^2 - 2x = 0 \end{cases}$$

$$\begin{cases} x + \lambda x - \lambda = 0 \\ y = 0 \\ x^2 - 2x = 0 \end{cases}$$

$$\begin{cases} x + \lambda x - \lambda = 0 \\ \frac{4\lambda}{4} = -\frac{1}{4} \\ x^2 + 4y^2 - 2x = 0 \end{cases}$$

$$\begin{cases} x + \lambda x - \lambda = 0 \\ y = 0 \\ x(x-2) = 0 \end{cases}$$

$$\begin{cases} x - \frac{1}{4}x + \frac{1}{4} = 0 \\ \lambda = -\frac{1}{4} \\ x^2 + 4y^2 - 2x = 0 \end{cases}$$

$$\begin{cases} x + \lambda x - \lambda = 0 \\ y = 0 \\ x = 0 \end{cases}$$

$$\begin{cases} \frac{4x - x + 1}{4} = 0 \\ \lambda = -\frac{1}{4} \\ x^2 + 4y^2 - 2x = 0 \end{cases}$$

$$\begin{cases} x + \lambda x - \lambda = 0 \\ y = 0 \\ x - 2 = 0 \end{cases}$$

$$\begin{cases} \lambda = 0 \\ y = 0 \\ x = 0 \end{cases}$$

$$(0, 0, 0)$$

$$\begin{cases} 2 + 2\lambda - \lambda = 0 \\ y = 0 \\ x = 2 \end{cases}$$

$$\begin{cases} \lambda + 2 = 0 \\ y = 0 \\ x = 2 \end{cases}$$

$$\begin{cases} \lambda = -2 \\ y = 0 \\ x = 2 \end{cases}$$

$$(2, 0, -2)$$

$$\begin{cases} 3x + 1 = 0 \\ x = -\frac{1}{4} \\ x^2 + 4y^2 - 2x = 0 \end{cases}$$

$$\begin{cases} \frac{2}{3}x = -\frac{1}{3} \\ x = -\frac{1}{4} \\ (-\frac{1}{3})^2 + 4y^2 - 2(-\frac{1}{3}) = 0 \end{cases}$$

$$\begin{cases} x = -\frac{1}{3} \\ \lambda = -\frac{1}{4} \\ \frac{1}{9} + 4y^2 + \frac{2}{3} = 0 \end{cases}$$

$$\begin{cases} x = -\frac{1}{3} \\ \lambda = -\frac{1}{4} \\ 1 + 36y^2 + 6 = 0 \end{cases}$$

$$\begin{cases} x = -\frac{1}{3} \\ \lambda = -\frac{1}{4} \\ 36y^2 + 7 = 0 \\ \text{I.M.P.} \end{cases}$$

I PUNTI STAZIONARI SONO:

$$(0,0,0) \quad (2,0,-2)$$

SCRIVIAMO L'HESSIANO

$$H(x,y,\lambda) = \begin{vmatrix} 0 & 2x-2 & 8y \\ 2x-2 & 2+2\lambda & 0 \\ 8y & 0 & 2+8\lambda \end{vmatrix}$$

$$H(0,0,0) = \begin{vmatrix} 0 & -2 & 0 \\ -2 & 2 & 0 \\ 0 & 0 & 2 \end{vmatrix}$$

IL DETERMINANTE SI RISOLVE AFFIANCANDO  
ALLE TRE COLONNE LA PRIMA E LA SECONDA

$$\begin{array}{ccc|cc} 0 & -2 & 0 & 0 & -2 \\ -2 & 2 & 0 & -2 & 2 \\ 0 & 0 & 2 & 0 & 0 \end{array} = 0+0+0-8+0-0=-8<0$$

SI ESEGUISCONO I PRODOTTI IN DIAGONALE  
DALL'ALTO IN BASSO DA SINISTRA A SINISTRA  
DESTRA CON SEGNO UGUALE DEL RISULTATO  
E GLI ALTRI CON SEGNO CA MBIATO

$$(0,0) \text{ E' UN MINIMO VINCOLATO}$$

$$H(2,0,-2) = \begin{vmatrix} 0 & 2 & 0 \\ 2 & -2 & 0 \\ 0 & 0 & -14 \end{vmatrix}$$

$$\begin{array}{ccc|cc} 0 & 2 & 0 & 2 & \\ 2 & -2 & 0 & -2 & -2 \\ 0 & 0 & -14 & 0 & 0 \end{array} = 0+0+0+56-0-0=56>0$$

(2,0) E' UN MASSIMO VINCOLATO